STAT 2593

Lecture 009 - Independence

Dylan Spicker

Independence

Learning Objectives

1. Understand the concepts of independence and dependence, intuitively and mathematically.

2. Understand the concepts of mutual independence.

3. Understand the properties of independence and dependence.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.
  - Specifically, if $P(A \cap B) = P(A)P(B)$, then A and B are said to be **independent**.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.
  - Specifically, if $P(A \cap B) = P(A)P(B)$, then A and B are said to be **independent**.
  - If A and B are not independent, we say that they are **dependent**.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.
  - Specifically, if $P(A \cap B) = P(A)P(B)$, then A and B are said to be **independent**.
  - If A and B are not independent, we say that they are **dependent**.
  - We write $A \perp B$ for independence, and $A \not\perp B$ for dependence.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.
  - Specifically, if $P(A \cap B) = P(A)P(B)$, then A and B are said to be **independent**.
  - If A and B are not independent, we say that they are **dependent**.
  - We write $A \perp B$ for independence, and $A \not\perp B$ for dependence.
  - If $A \perp B$, then $P(A|B) = P(A)$.

Independence

- With conditional probabilities, we saw that sometimes knowing an event B impacts our knowledge about A.
- When this *does not* happen, we say that A and B are independent.
  - Specifically, if $P(A \cap B) = P(A)P(B)$, then A and B are said to be **independent**.
  - If A and B are not independent, we say that they are **dependent**.
  - We write $A \perp B$ for independence, and $A \not\perp B$ for dependence.
  - If $A \perp B$, then $P(A|B) = P(A)$.
- Intuitively, independence means that knowledge of A tells us nothing of B (and vice versa).

Properties of Independence

- If $A \perp B$ then we also know that:

Properties of Independence

- If $A \perp B$ then we also know that:
  - $B \perp A$;

Properties of Independence

- If $A \perp B$ then we also know that:
  - $B \perp A$;
  - $A^C \perp B$;

Properties of Independence

- If $A \perp B$ then we also know that:
  - $B \perp A$;
  - $A^C \perp B$;
  - $A \perp B^C$;

Properties of Independence

- If $A \perp B$ then we also know that:
  - $B \perp A$;
  - $A^C \perp B$;
  - $A \perp B^C$;
  - $A^C \perp B^C$

Mutual Independence

- If we have many events, $A_1, \ldots, A_k$ we can define **mutual independence**.

Mutual Independence

- If we have many events, $A_1, \ldots, A_k$ we can define **mutual independence**.
- The events are mutually independent, if, for **every subset** of these events of **every size** between $\ell = 2$ and $\ell = k$,

$$P(\cap_{j=1}^{\ell} A_j) = \prod_{j=1}^{\ell} P(A_j).$$

Mutual Independence

- If we have many events, $A_1, \ldots, A_k$ we can define **mutual independence**.
- The events are mutually independent, if, for **every subset** of these events of **every size** between $\ell = 2$ and $\ell = k$,

$$P(\cap_{j=1}^{\ell} A_j) = \prod_{j=1}^{\ell} P(A_j).$$

- That is, every subset of events requires the multiplicative property.

Mutual Independence

- If we have many events, $A_1, \ldots, A_k$ we can define **mutual independence**.
- The events are mutually independent, if, for **every subset** of these events of **every size** between $\ell = 2$ and $\ell = k$,

$$P(\cap_{j=1}^{\ell} A_j) = \prod_{j=1}^{\ell} P(A_j).$$

- That is, every subset of events requires the multiplicative property.
- Note, if $A \perp B$ and $B \perp C$, then it is not the case that $A \perp C$.

Mutual Independence

- If we have many events, $A_1, \ldots, A_k$ we can define **mutual independence**.
- The events are mutually independent, if, for **every subset** of these events of **every size** between $\ell = 2$ and $\ell = k$,

$$P(\cap_{j=1}^{\ell} A_j) = \prod_{j=1}^{\ell} P(A_j).$$

- That is, every subset of events requires the multiplicative property.
- Note, if $A \perp B$ and $B \perp C$, then it is not the case that $A \perp C$.
  - Similarly, if $A \perp B$, $B \perp C$, and $A \perp C$, it may not be the case that A, B, C are mutually independent.

Mutually Exclusive

- If two events **cannot** happen simultaneously, we say that they are **mutually exclusive**.

Mutually Exclusive

- If two events **cannot** happen simultaneously, we say that they are **mutually exclusive**.

- Events which are **mutually exclusive** are *always* dependent.

Mutually Exclusive

- If two events **cannot** happen simultaneously, we say that they are **mutually exclusive**.
- Events which are **mutually exclusive** are *always* dependent.
  - Why?

Summary

- Independence codifies the idea that some events give no information about one another.
- Independence can be defined through joint or conditional probabilities.
- Independence implies independence of many derived quantities.
- Mutual independence is a stronger form of independence, when many events exist.
- Mutually exclusive events are not independent.